prDeep: Robust Phase Retrieval with a Flexible Deep Network

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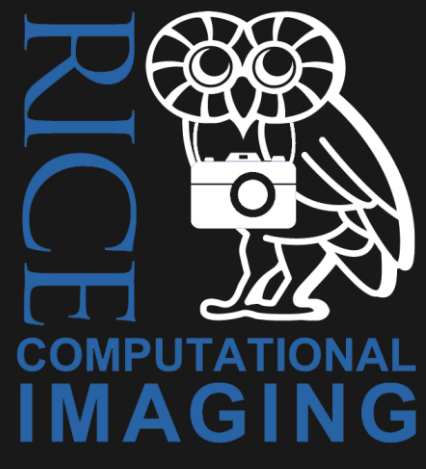


Phase Retrieval (PR) Applications

Given $y = |\boldsymbol{A}x| + w$, solve for x

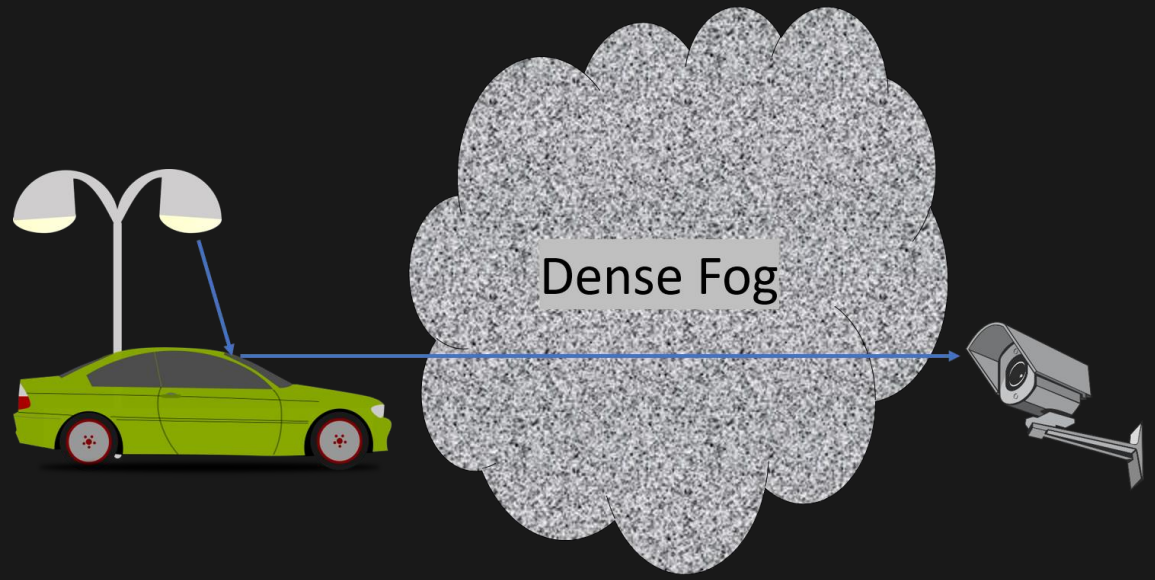
Plays central role in

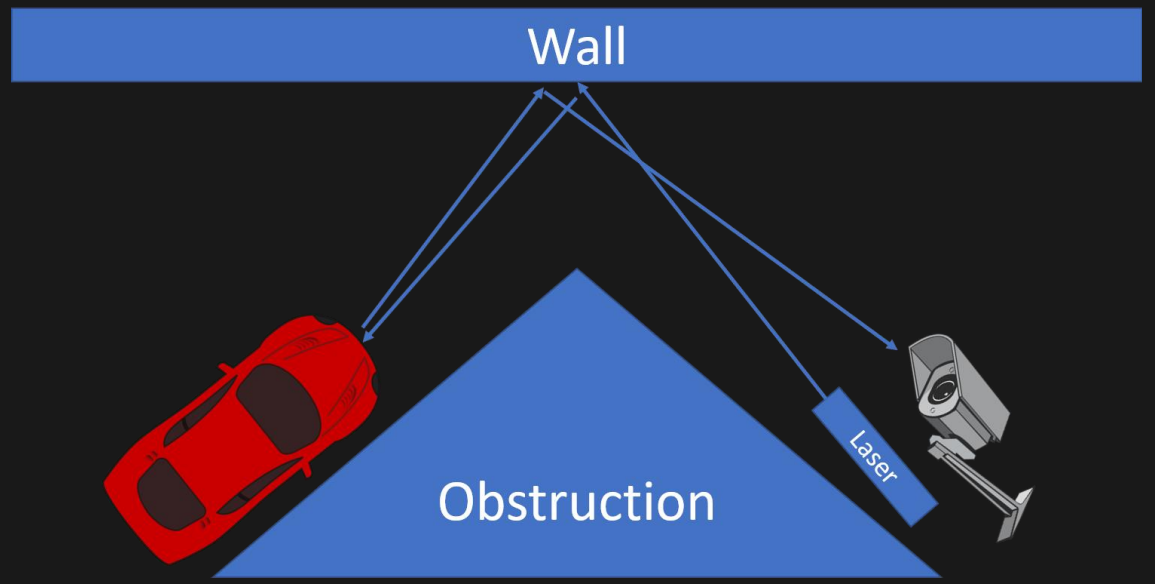
- Astronomy
- Microscopy
- Crystallography
- X-ray imaging

Interested in special case: $\boldsymbol{A}$ is DFT matrix, x is real and nonnegative

- Imaging through scattering media [Katz et al. 2014]
- Imaging around corners [Rangarajan et al. 2018]

Imaging Through Scattering Media and Around Corners





- How it works:
  - Estimate autocorrelation function of target, $x \star x$
  - Use relation $F(x \star x) = |Fx|^2$ to compute $|Fx|^2$
  - Reconstruct x from $|Fx|^2$ with PR
- Very few photons sent into system make it to detector
  - Model (innacurately) as Poisson noise on $|Fx|^2$

Phase Retrieval Algorithms Struggle with Noise

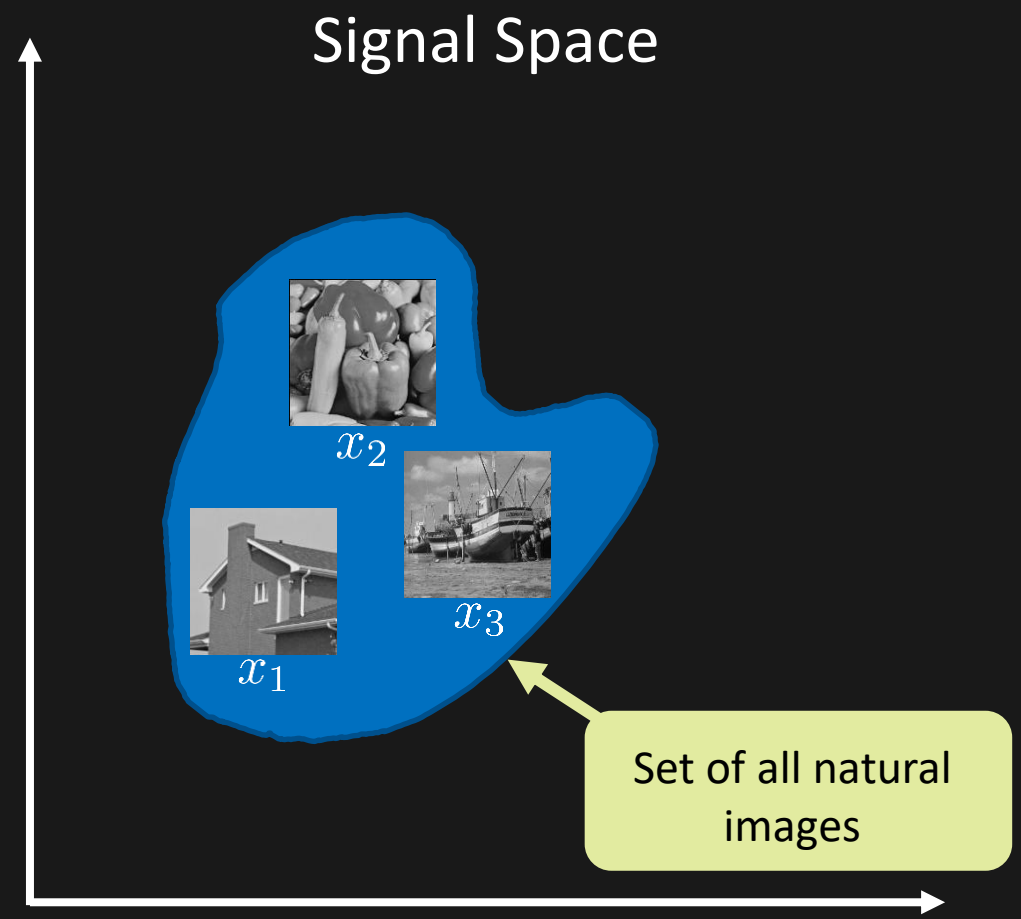
Given $y = |\boldsymbol{A}x| + w$, solve for x

- Standard, alternating projection methods
  - Not robust to noise
- Optimization-based methods
  - Robust to noise by imposing priors, e.g., sparsity or smoothness
  - Typically require Gaussian or coded diffraction pattern measurements $\boldsymbol{A}$
- Plug and Play ADMM for PR [Venkatakrishnan et al. 2013, Heide et al. 2016]
  - Robust to noise by imposing priors with denoisers, e.g., BM3D
  - Handles Fourier measurements
  - Computationally expensive
  - No public implementation

This work: Apply deep learning to develop fast and noise-robust phase retrieval algorithms that can handle arbitrary measurements, including Fourier

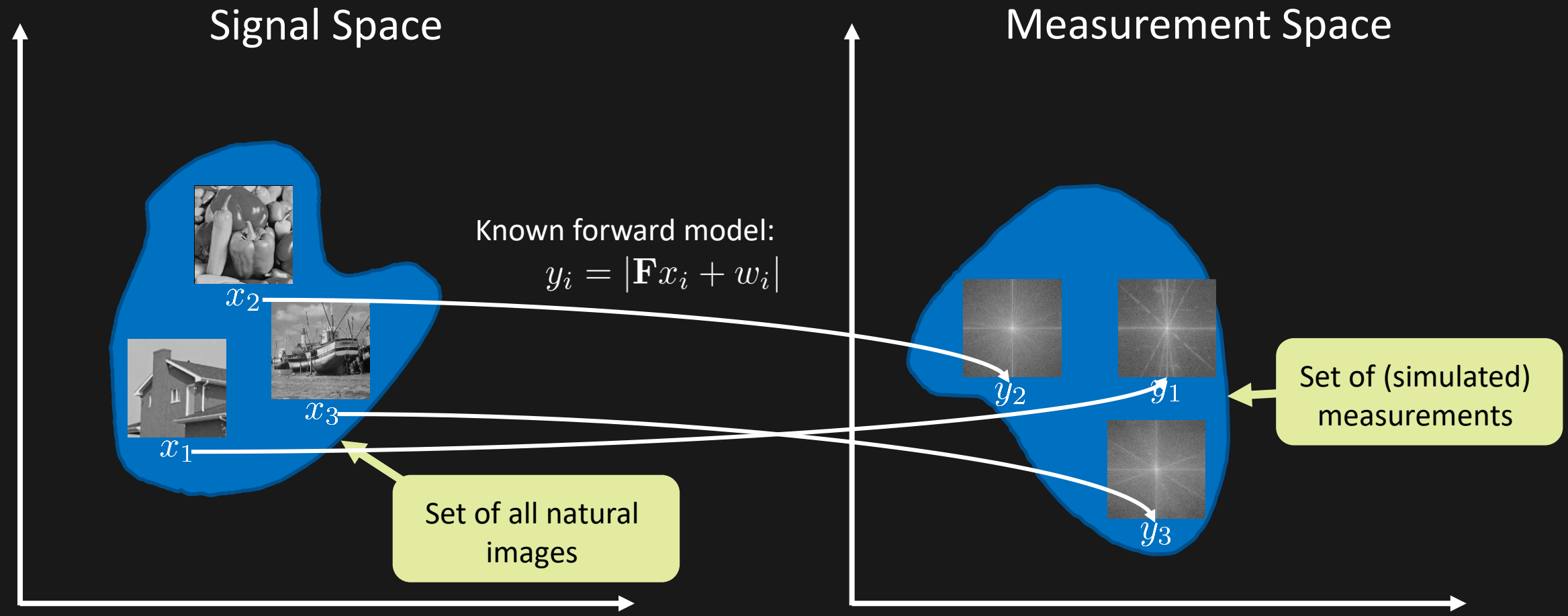
Standard Deep Learning Approach

- Training data: x_1, x_2, …



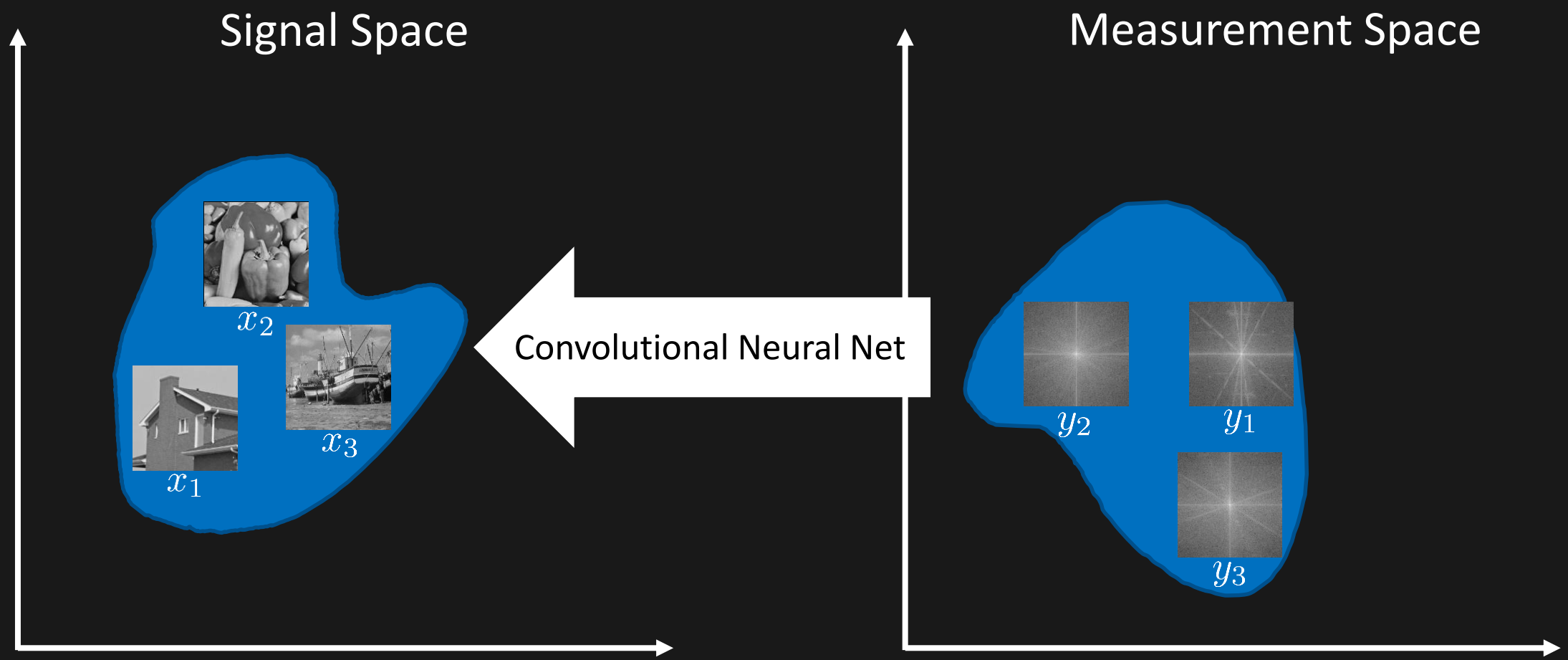
Standard Deep Learning Approach

- Training data: x_1, x_2, …
- Generate measurements with known forward model: $y_i = f(x_i)$



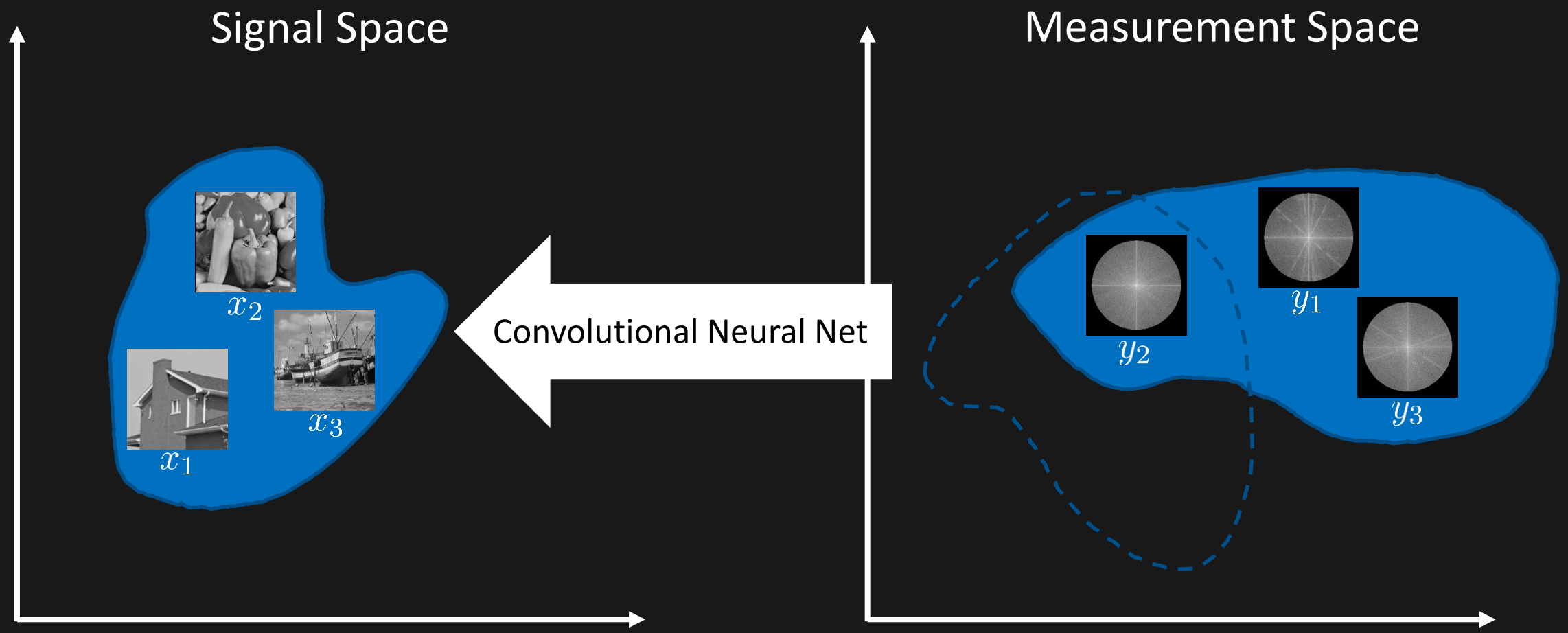
Standard Deep Learning Approach

- **Learn inverse mapping** from measurement space to signal space



Problem with Standard Deep Learning Approach

- If measurement model changes, then network is useless



Proposed: CNN as Part of an Algorithm

- Start with iterative algorithm that imposes priors using image denoiser
  - Denoisers as projections onto the set of natural images

- Train a neural network to denoise images

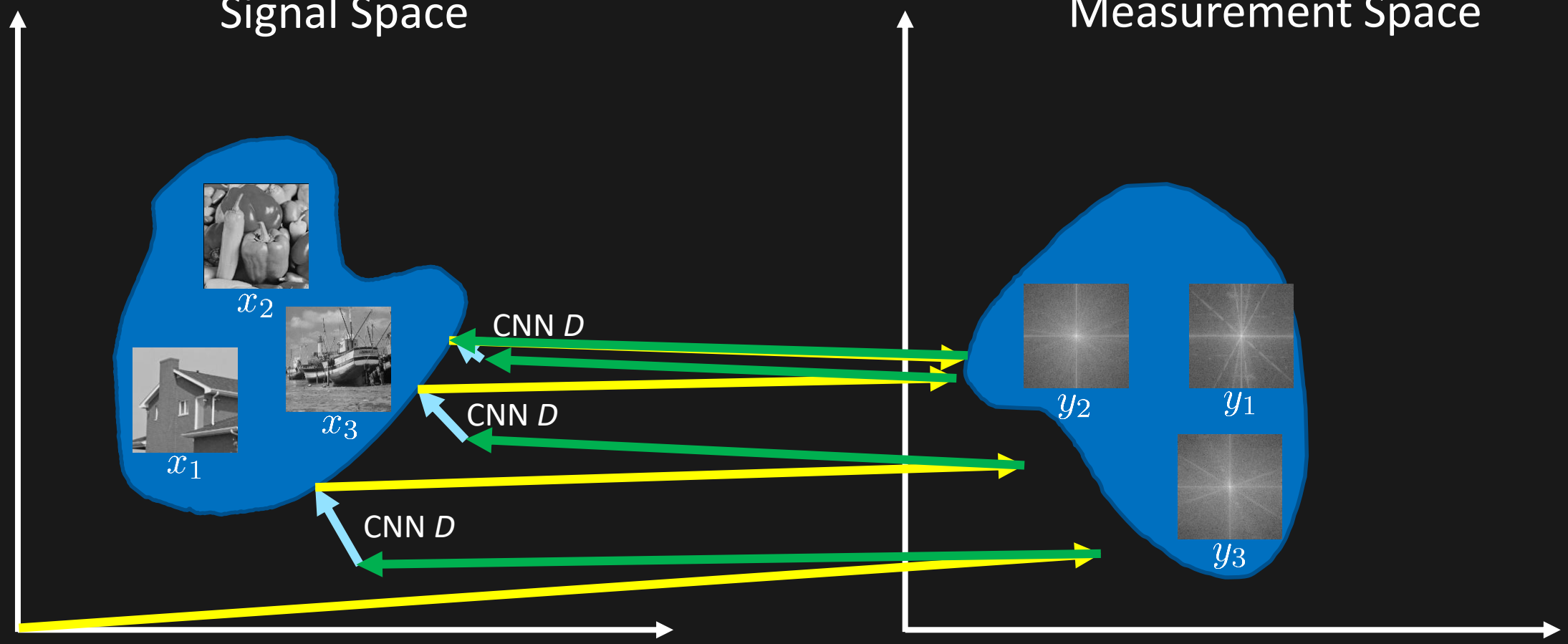
- Replace the image denoiser with a neural network

Proposed: CNN as Part of an Algorithm, Basic Example

Compute predicted measurements

Gradient step to satisfy measurements

Denoising CNN step to satisfy prior



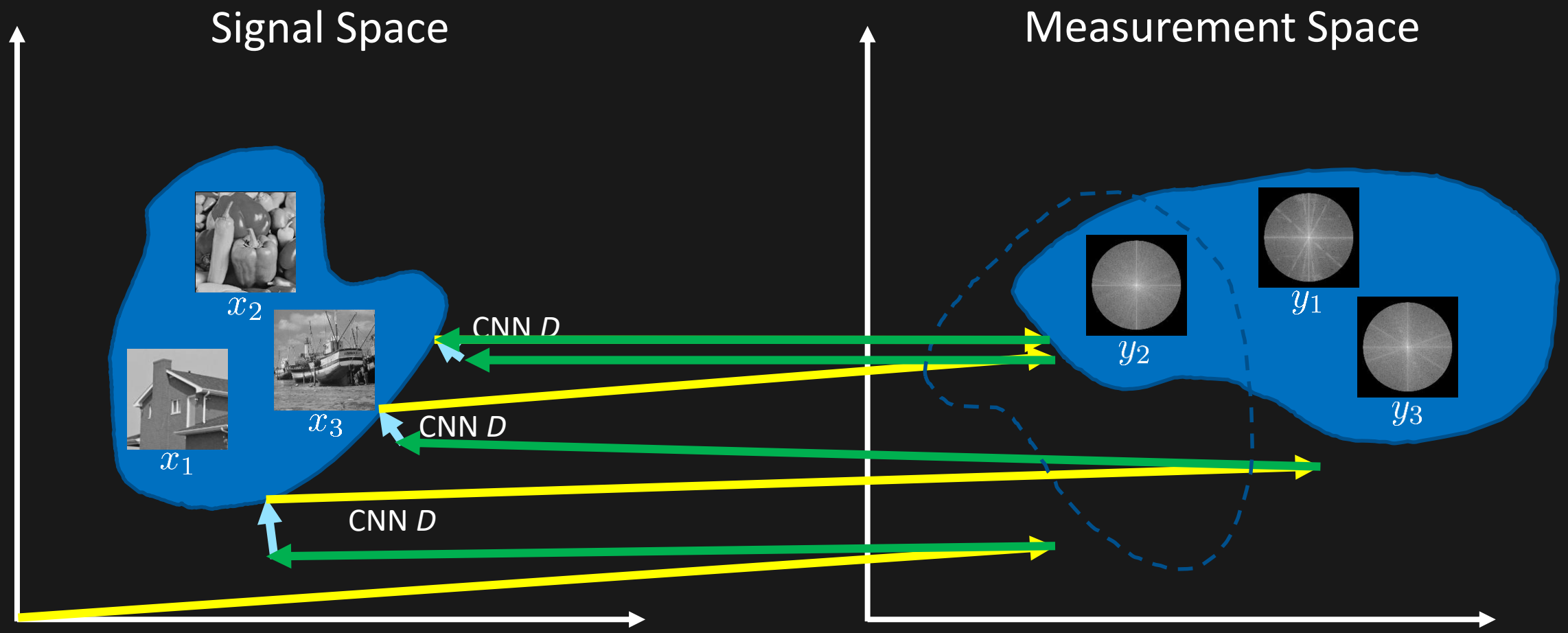
Proposed: CNN as Part of an Algorithm, Basic Example

Compute predicted measurements

Gradient step to satisfy measurements

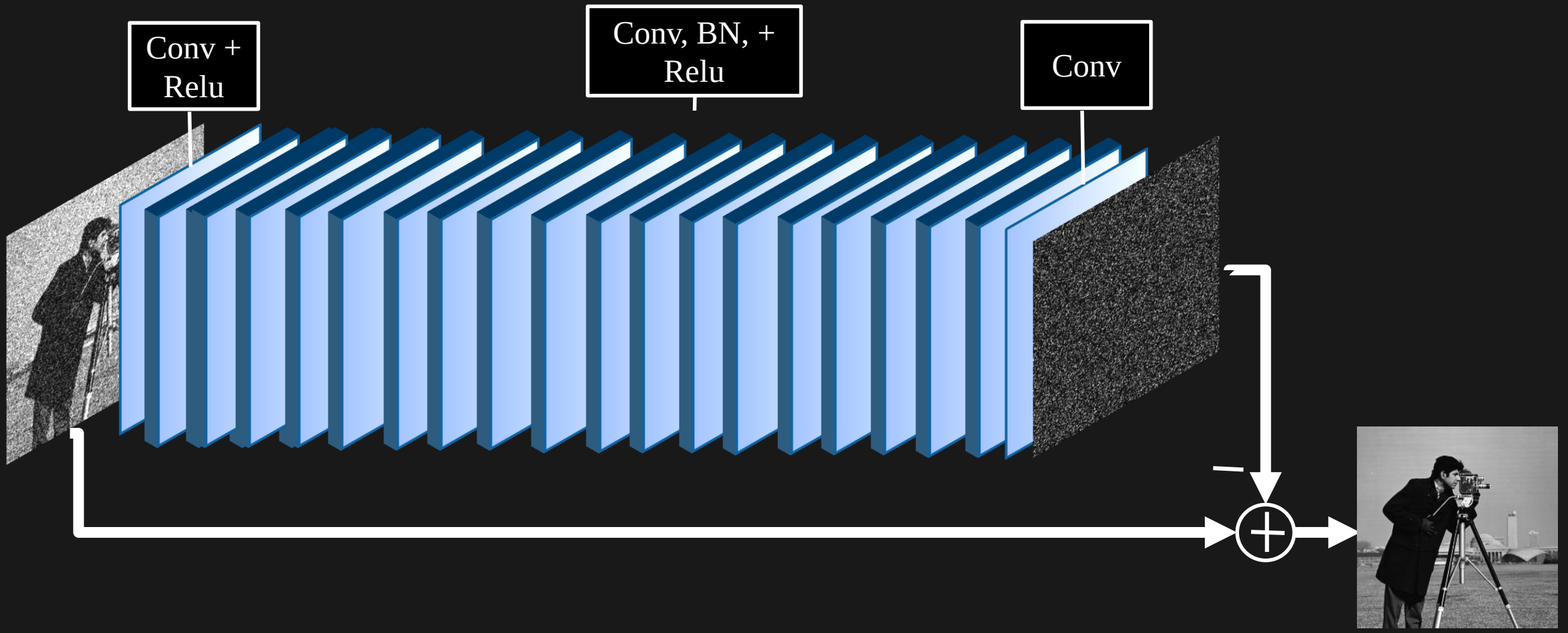
Denoising CNN step to satisfy prior

Works even if measurement model changes



Step 1: Train a Neural Network to Denoise Noisy Images

- We use the DnCNN denoiser [Zhang et al. 2017]
- Train offline with 400 images divided into 300,000 50x50 image patches (3 hours)



Step 2: Set up an Alg that Imposes Priors with Denoisers

- L^2 amplitude loss function
- RED: Regularization by Denoising [Romano et al. 2017]

$$\arg\min_x \underbrace{\| y - |\mathbf{F}x| \|^2}_{\text{Analytic subgradient}} + \underbrace{\lambda x^t (x - D(x))}_{\text{Has proximal mapping}}$$

- Can solve with many different methods. We use FASTA [Goldstein et al. 2014]

- Need to initialize non-convex problem. We use HIO algorithm [Fienup 82]

Simulation: 4x Fourier Measurements with Poisson Noise

Hybrid Input Output (HIO) (40 sec)

prDeep (40 + 35 sec)

Incorporating training data and deep learning makes PR algorithms far more robust

Simulation: 4x Fourier Measurements with Poisson Noise

BM3D-ADMM (40 + 104 sec)

prDeep (40 + 35 sec)

Incorporating training data and deep learning makes PR algorithms far more robust

Summary

- PR problem shows up in numerous applications
- Fourier PR problem is especially important
  - Image through scattering media and around corners
- These applications are photon limited
  - Lots of measurement noise
- This work: Use learned priors for noise robust phase retrieval - prDeep
  - Setup algorithm that imposes prior with denoiser
  - Replace denoiser with CNN

- Poster: #164 tonight in Hall B
- Code: https://github.com/ricedsp/prDeep

Imaging with Speckle Correlations [Bertolotti et al. 2012, Katz et al. 2014]

- Capture a single image of unknown object (O) with unknown speckle PSF (S)
- Convolution model:

$$I = O * S$$

- Assume autocorrelation of S is identity
- The autocorrelation of *I* approximates the autocorrelation of *O*

$$I \star I = (O * S) \star (O * S) = (O \star O) * (S \star S) \approx O \star O$$

- *O* is related to its autocorrelation through the Fourier transform

$$F(O \star O) = |F(O)|\text{^}2$$

- Reconstruct *O* by solving phase retrieval problem