prDeep: Robust Phase Retrieval with a Flexible Deep Network

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Phase Retrieval (PR) Applications

Given $y = |Ax| + w$, solve for $x$

Plays central role in

– Astronomy
– Microscopy
– Crystallography
– X-ray imaging

Interested in special case: $A$ is DFT matrix, $x$ is real and nonnegative

– Imaging through scattering media [Katz et al. 2014]
– Imaging around corners [Rangarajan et al. 2018]
• How it works:
  – Estimate autocorrelation function of target, $x \ast x$
  – Use relation $F(x \ast x) = |Fx|^2$ to compute $|Fx|^2$
  – Reconstruct $x$ from $|Fx|^2$ with PR

• Very few photons sent into system make it to detector
  – Model (inaccurately) as Poisson noise on $|Fx|^2$
Phase Retrieval Algorithms Struggle with Noise

Given $y = |Ax| + w$, solve for $x$

- Standard, alternating projection methods
  - Not robust to noise
- Optimization-based methods
  - Robust to noise by imposing priors, e.g., sparsity or smoothness
  - Typically require Gaussian or coded diffraction pattern measurements $A$
- Plug and Play ADMM for PR [Venkatakrishnan et al. 2013, Heide et al. 2016]
  - Robust to noise by imposing priors with denoisers, e.g., BM3D
  - Handles Fourier measurements
  - Computationally expensive
  - No public implementation

This work: Apply deep learning to develop fast and noise-robust phase retrieval algorithms that can handle arbitrary measurements, including Fourier
Standard Deep Learning Approach

- Training data: $x_1, x_2, \ldots$

Signal Space

Set of all natural images
Standard Deep Learning Approach

- Training data: $x_1, x_2, \ldots$
- Generate measurements with known forward model: $y_i = f(x_i)$
Learn inverse mapping from measurement space to signal space

Standard Deep Learning Approach

Signal Space

Measurement Space

Convolutional Neural Net

$X_1$, $X_2$, $X_3$ are measurement samples in the measurement space.

$Y_1$, $Y_2$, $Y_3$ are corresponding signal samples in the signal space.
Problem with Standard Deep Learning Approach

- If measurement model changes, then network is useless
Proposed: CNN as Part of an Algorithm

• Start with iterative algorithm that imposes priors using image denoiser
  – Denoisers as projections onto the set of natural images

• Train a neural network to denoise images

• Replace the image denoiser with a neural network
Proposed: CNN as Part of an Algorithm, Basic Example

- Compute predicted measurements
- Gradient step to satisfy measurements
- Denoising CNN step to satisfy prior

Signal Space

Measurement Space
Proposed: CNN as Part of an Algorithm, Basic Example

- Compute predicted measurements
- Gradient step to satisfy measurements
- Denoising CNN step to satisfy prior

Works even if measurement model changes
Step 1: Train a Neural Network to Denoise Noisy Images

- We use the **DnCNN denoiser** [Zhang et al. 2017]
- Train offline with 400 images divided into 300,000 50x50 image patches (3 hours)
Step 2: Set up an Alg that Imposes Priors with Denoisers

- L² amplitude loss function
- RED: Regularization by Denoising [Romano et al. 2017]

\[
\text{arg min}_x \| y - |Fx| \|^2 + \lambda x^t (x - D(x))
\]

- Can solve with many different methods. We use FASTA [Goldstein et al. 2014]
- Need to initialize non-convex problem. We use HIO algorithm [Fienup 82]
Simulation: 4x Fourier Measurements with Poisson Noise

Hybrid Input Output (HIO) (40 sec)

prDeep (40 + 35 sec)

Incorporating training data and deep learning makes PR algorithms far more robust
Simulation: 4x Fourier Measurements with Poisson Noise

BM3D-ADMM (40 + 104 sec)  prDeep (40 + 35 sec)

Incorporating training data and deep learning makes PR algorithms far more robust
Summary

• PR problem shows up in numerous applications

• Fourier PR problem is especially important
  – Image through scattering media and around corners

• These applications are photon limited
  – Lots of measurement noise

• This work: Use learned priors for noise robust phase retrieval - prDeep
  – Setup algorithm that imposes prior with denoiser
  – Replace denoiser with CNN

• Poster: #164 tonight in Hall B
• Code: https://github.com/ricedsp/prDeep

• Capture a **single image** of unknown object (O) with unknown speckle PSF (S)
• Convolution model:

\[ I = O \ast S \]

• Assume autocorrelation of S is identity
• The autocorrelation of I approximates the autocorrelation of O

\[ I \ast I = (O \ast S) \ast (O \ast S) = (O \ast O) \ast (S \ast S) \approx O \ast O \]

• O is related to its autocorrelation through the Fourier transform

\[ F(O \ast O) = |F(O)|^2 \]

• Reconstruct O by solving phase retrieval problem